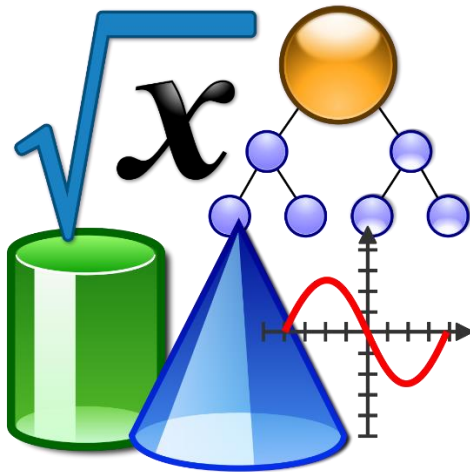


NPS Learning in Place Algebra II



Name: _____ School: _____ Teacher: _____

May 18 – June 5

Week 1	<ul style="list-style-type: none">• Functions and Transformations
Week 2	<ul style="list-style-type: none">• Rational Functions
Week 3	<ul style="list-style-type: none">• Curve of Best Fit• Direct/Inverse/Joint Variation

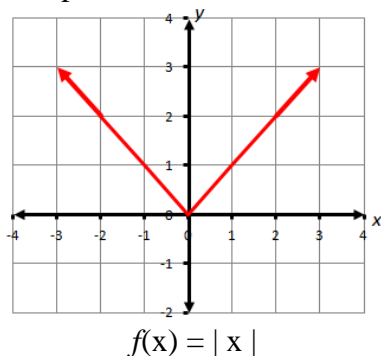
Week 1

Day 1 Absolute Value Functions

How do you graph and write absolute value functions?

Vocabulary

The parent function of an **absolute value function** is defined as $f(x) = |x|$.



The highest or lowest point on the graph of an absolute value function is called the **vertex of an absolute function graph**.

A **transformation** changes a graph's size, shape, position, or orientation.

A **translation** is a transformation that shifts a graph horizontally and/or vertically, but does not change its size, shape, or orientation.

When $a = -1$, the graph of $y = a|x|$ is a **reflection** in the x -axis of the graph of $y = |x|$.

Transformations

$$f(x) = -a|x - h| + k$$

Reflection
across the
 x -axis

Vertical Stretch
 $a > 1$
(makes it narrower)
OR
Vertical Compression
 $0 < a < 1$
(makes it wider)

Vertical
Translation

Horizontal
Translation
(opposite of h)

Remember that (h, k) is your vertex
“ a ” may represent as a slope

EXAMPLE:

Graph a function of the form $y = a|x - h| + k$

Graph $y = 3|x + 2| - 1$. Compare the graph with the graph of $y = |x|$.

Solution

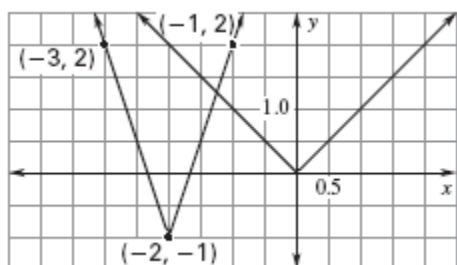
The graph of $y = a|x - h| + k$ is the graph of $y = |x|$ translated h units horizontally and k units vertically with its vertex at (h, k) . The factor a stretches or shrinks the graph.

STEP 1 Identify and plot the vertex, $(h, k) = (-2, -1)$.

STEP 2 Plot another point $(-1, 2)$ on the graph. Use symmetry to plot a third point $(-3, 2)$.

STEP 3 Connect the points with a V-shaped graph.

STEP 4 Compare with $y = |x|$. The graph of $y = 3|x + 2| - 1$ is the graph of $y = |x|$ first stretched vertically by a factor of 3, then translated left 2 units and down 1 unit.



$$a = 3, h = -2, k = -1$$

Vertex: $(-2, -1)$

Reflected: No

Horizontal translation: 2 units left

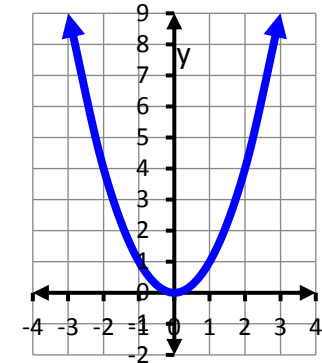
Vertical translation: 1 unit down

Vertical stretch/compression:
stretched vertically by a factor of 3

Day 2 Quadratic Functions

How do you graph quadratic functions?

The parent function of a quadratic function is defined as $f(x) = x^2$.



$$f(x) = x^2.$$

Transformations

$$f(x) = -a(x - h)^2 + k$$

Reflection
across the
x-axis

Vertical Stretch
 $a > 1$
(makes it narrower)
OR
Vertical Compression
 $0 < a < 1$
(makes it wider)

Horizontal
Translation
(opposite of h)

Vertical
Translation

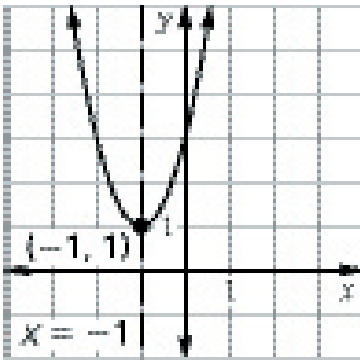
Remember that (h, k) is your vertex

The vertex form of a quadratic equation is given by $y = a(x - h)^2 + k$.

Graphing Quadratic Functions in $y = a(x - h)^2 + k$ form

Example:

Given the graph of $y = 2(x + 1)^2 + 1$, compare the graph from the graph of $f(x) = x^2$.



Solution: The graph of $y = a(x - h)^2 + k$ is the graph of $y = x^2$ translated h units horizontally and k units vertically with its vertex at (h, k) . The factor a stretches or shrinks the graph.

The graph of $y = 2(x + 1)^2 + 1$ is the graph of $y = x^2$ first stretched vertically by a factor of 2, then translated left 1 unit and up 1 unit.

Step 1 Create table of values

x	$y = 2(x + 1)^2 + 1$
-2	3
-1	1
0	3
1	9

$$a = 2, h = -1, k = 1$$

Vertex: $(-1, 1)$

Reflected: No

Horizontal translation: 1 unit left

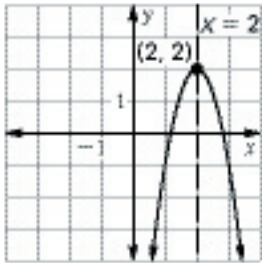
Vertical translation: 1 unit up

Vertical stretch/compression: stretched vertically by a factor of 2

Step 2. Plot the points and connect them in a smooth continuous curve.

Practice: Given the graph $f(x) = a(x - h)^2 + k$, compare the graph from the graph of $f(x) = x^2$.

1. $y = -3(x - 2)^2 + 2$



$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$

Vertex:

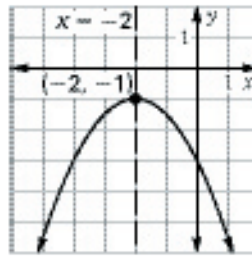
Reflected:

Horizontal translation:

Vertical translation:

Vertical stretch/compression:

2. $y = -\frac{1}{2}(x + 2)^2 - 1$



$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$

Vertex:

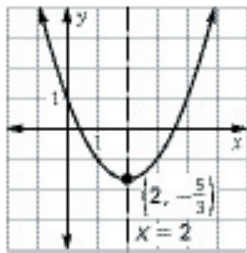
Reflected:

Horizontal translation:

Vertical translation:

Vertical stretch/compression:

3. $y = \frac{2}{3}(x - 2)^2 - \frac{5}{3}$



$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$

Vertex:

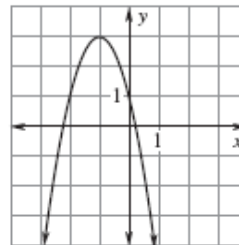
Reflected:

Horizontal translation:

Vertical translation:

Vertical stretch/compression:

4. $y = -2(x + 1)^2 + 3$



$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$

Vertex:

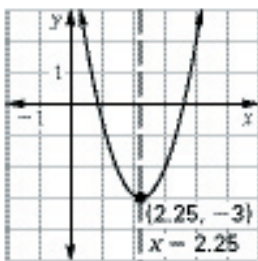
Reflected:

Horizontal translation:

Vertical translation:

Vertical stretch/compression:

5. $y = 1.6(x - 2.25)^2 - 3$



$a = \underline{\quad}, h = \underline{\quad}, k = \underline{\quad}$

Vertex:

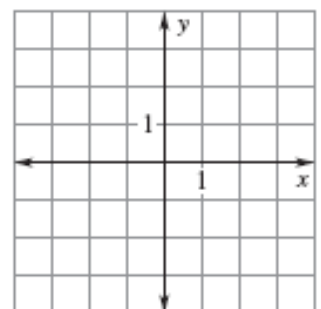
Reflected:

Horizontal translation:

Vertical translation:

Vertical stretch/compression:

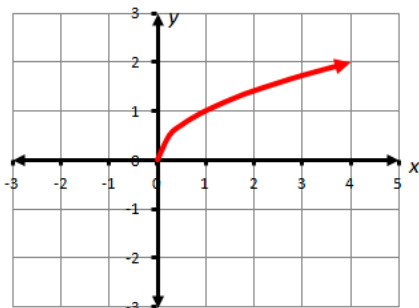
6. Graph $y = -(x - 1)^2 + 2$



Day 3

Graph square root and cube root functions.

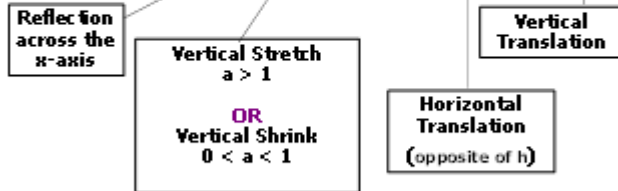
The parent function of a square root function is defined as $f(x) = \sqrt{x}$.



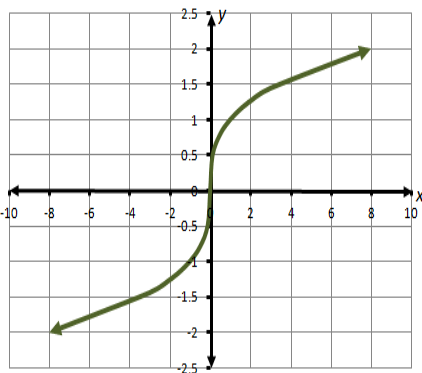
$f(x) = \sqrt{x}$

Transformations

$$f(x) = -a\sqrt{x-h} + k$$



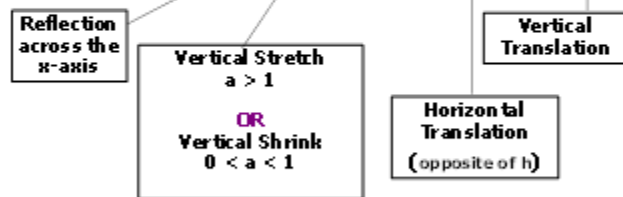
The parent function of a cube root function is defined as $f(x) = \sqrt[3]{x}$.



$f(x) = \sqrt[3]{x}$

Transformations

$$f(x) = -a\sqrt[3]{x-h} + k$$



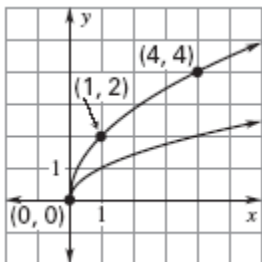
EXAMPLE 1

Graph a square root function

Graph $y = 2\sqrt{x}$. Compare the graph with the graph of $y = \sqrt{x}$.

Solution:

Make a table of values and sketch the graph.



x	0	1	2	3	4
$y = 2\sqrt{x}$	0	2	2.83	3.46	4

The graph of $y = 2\sqrt{x}$ is vertical stretch of the parent graph $y = \sqrt{x}$.

$a = 2, h = 0, k = 0$

End point: $(0,0)$

Reflected: No

Horizontal translation: None

Vertical translation: None

Vertical stretch/compression:

stretched vertically by a factor of 2

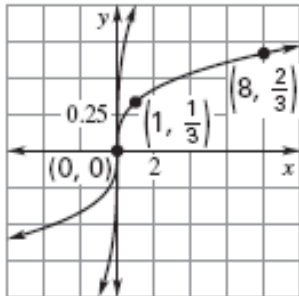
EXAMPLE 2

Graph a cube root function

Graph $y = \frac{1}{3}\sqrt[3]{x}$. Compare the graph with the graph of $y = \sqrt[3]{x}$.

Solution

Make a table of values and sketch the graph.



x	-2	-1	0	1	2	8
$y = \frac{1}{3}\sqrt[3]{x}$	-0.42	-0.33	0	0.33	0.42	0.67

The graph of $y = \frac{1}{3}\sqrt[3]{x}$ is vertical shrink of the parent graph $y = \sqrt[3]{x}$ by a factor of $\frac{1}{3}$.

$a = \frac{1}{3}, h = 0, k = 0$ **Point of symmetry: (0,0)** **Reflected: No**
Horizontal translation: None **Vertical translation: None**
Vertical stretch/compression: vertical compression by a factor of $\frac{1}{3}$

Practice:

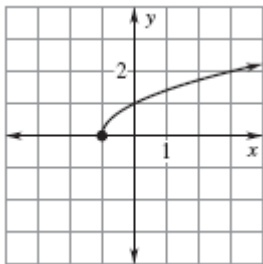
I. Match the function with each graph.

_____ 1. $f(x) = \sqrt{x} + 1$
 _____ 4. $f(x) = -\sqrt[3]{x+1}$

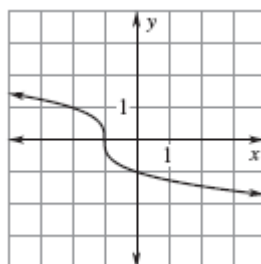
_____ 2. $f(x) = \sqrt{x+1}$
 _____ 5. $f(x) = \sqrt[3]{x+1}$

_____ 3. $f(x) = -\sqrt{x+1}$
 _____ 6. $f(x) = \sqrt[3]{x-1}$

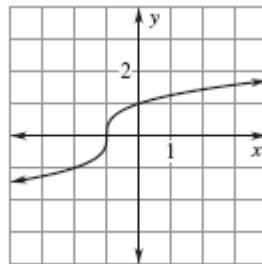
A.



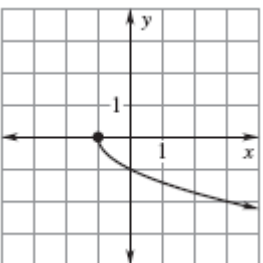
B.



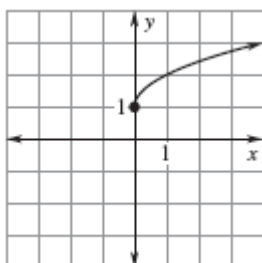
C.



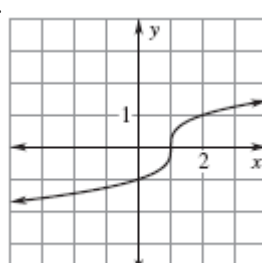
D.



E.

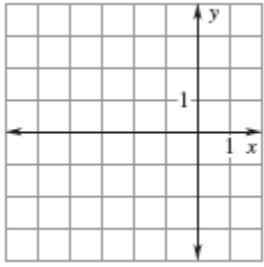


F.



II. Graph the function. Compare the graph with the graph of $y = \sqrt[3]{x}$.

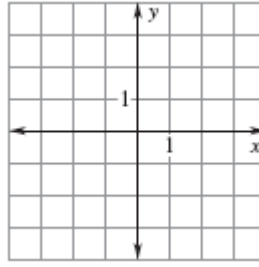
1. $f(x) = \sqrt{x+4} - 2$



$a = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}, k = \underline{\hspace{1cm}}$

End points: _____
 Reflected: _____
 Horizontal translation: _____
 Vertical translation: _____
 Vertical stretch/compression: _____

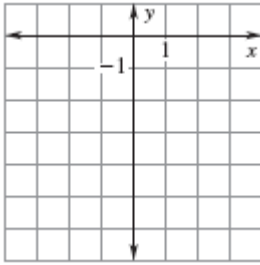
2. $f(x) = -3\sqrt{x} + 3$



$a = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}, k = \underline{\hspace{1cm}}$

End points: _____
 Reflected: _____
 Horizontal translation: _____
 Vertical translation: _____
 Vertical stretch/compression: _____

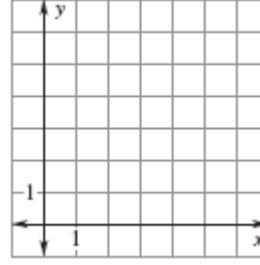
3. $f(x) = -2\sqrt[3]{x-3}$



$a = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}, k = \underline{\hspace{1cm}}$

Points of symmetry: _____
 Reflected: _____
 Horizontal translation: _____
 Vertical translation: _____
 Vertical stretch/compression: _____

4. $f(x) = \sqrt[3]{x-3} + 4$



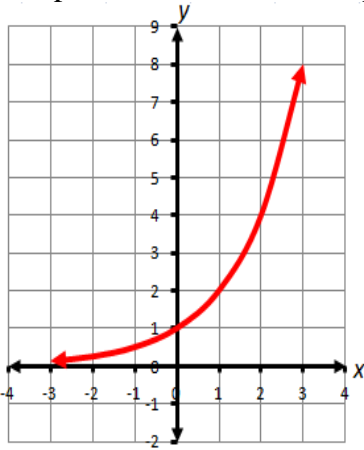
$a = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}, k = \underline{\hspace{1cm}}$

Points of symmetry: _____
 Reflected: _____
 Horizontal translation: _____
 Vertical translation: _____
 Vertical stretch/compression: _____

Day 4

EXPONENTIAL FUNCTIONS

The parent function of an exponential function is defined as $f(x) = b^x$.



$f(x) = b^x$

An **exponential function** has the form $y = ab^x$, where $a \neq 0$ and the base b is a positive number other than 1.

If $a > 0$ and $b > 1$, then the function $y = ab^x$ is an **exponential growth function**, and b is called the **growth factor**.

An **exponential decay function** has the form $y = ab^x$, where $a > 0$ and $0 < b < 1$.

The base b of an exponential decay function is called the **decay factor**.

EXAMPLE 1

Graph $y = b^x$ for $b > 1$

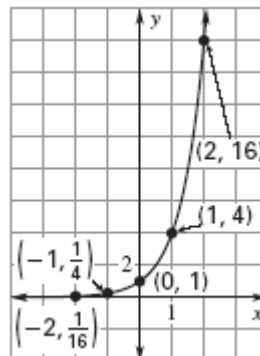
Graph $y = 4^x$.

Solution

STEP 1 Make a table of values.

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

STEP 2 Plot the points from the table.



STEP 3 Draw from *left to right*, a smooth curve that begins just above the x -axis, passes through the plotted points, and moves up to the right.

EXAMPLE 2

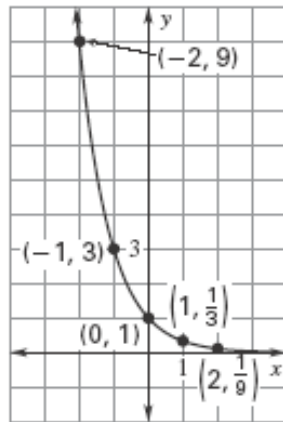
Graph $y = b^x$ for $0 < b < 1$

Graph $y = \left(\frac{1}{3}\right)^x$

Solution

STEP 1 Make a table of values.

x	-2	-1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



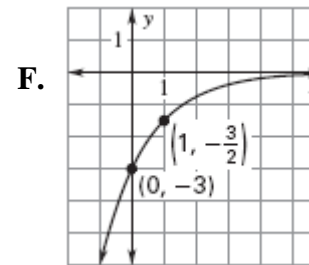
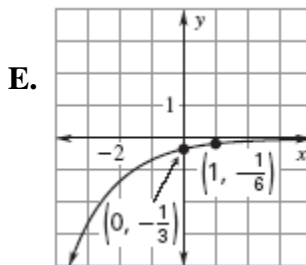
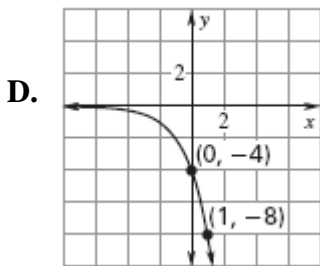
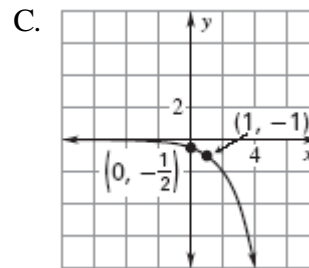
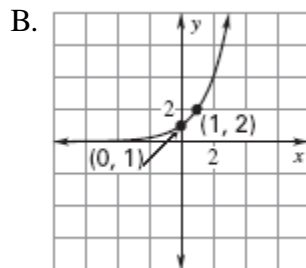
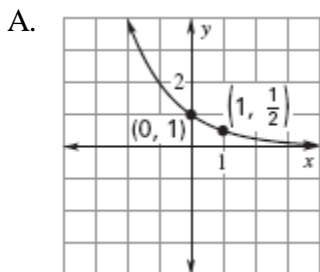
STEP 3 Draw from *right to left*, a smooth curve that begins just above the x -axis, passes through the plotted points, and moves up to the left.

STEP 2 Plot the points from the table.

Practice:

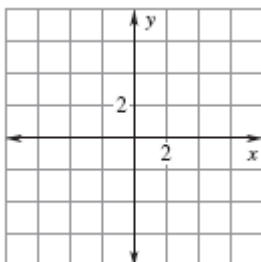
Match the function with its graph.

- _____ 1. $f(x) = (2)^x$ _____ 2. $f(x) = -\frac{1}{2}(2^x)$ _____ 3. $f(x) = -4(2^x)$
 _____ 4. $f(x) = \left(\frac{1}{2}\right)^x$ _____ 5. $f(x) = -3\left(\frac{1}{2}\right)^x$ _____ 6. $f(x) = -\frac{1}{3}\left(\frac{1}{2}\right)^x$

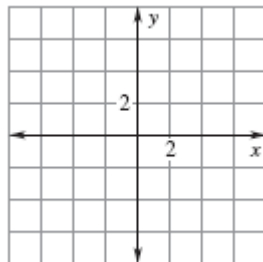


Graph the function.

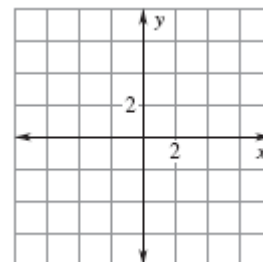
7. $f(x) = 4^x$



8. $f(x) = 2^{x-2} - 1$



9. $f(x) = -\left(\frac{2}{3}\right)^x + 3$



Day 5 Quiz on Graphing functions and its transformations.

1. What steps transform the graph $y = x^2$ to $y = 2(x+2)^2 - 5$?

- A. Shrink by 2, shifted 2 units left and 5 down
- B. Stretch by 5, shifted 5 units left and 2 down
- C. Stretch by 2, shifted 2 units left and 5 down
- D. Shrink by 5, shifted 2 units left and 2 down

2. Match the equation $y = -(x)^2 - 4$ to its description.

- A. Reflected over x and up 4
- B. Reflected over x and down 4
- C. Reflected over x and right 4
- D. Reflected over x and left 4

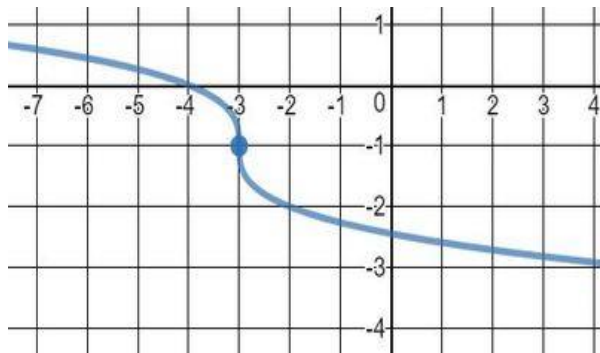
3. Which of the following describes the horizontal and vertical translations of the following functions: $f(x) = |x + 4| - 9$

- A. Left 4, up 9
- B. Left 4, Down 9
- C. Right 4, up 9
- D. Right 4, Down 9

4. Describe the transformation of $f(x) = \frac{1}{5}|x| - 4$ from the Absolute Value Parent Function.

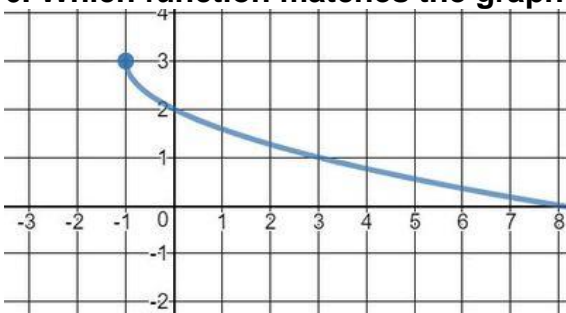
- A. Vertical Stretch, shifts up 4 units
- B. Vertical Stretch, shifts down 4 units
- C. Compresses wider, shifts up 4 units
- D. Compresses wider, shifts down 4 units

5. Which function matches the graph?



- A. $f(x) = -\sqrt[3]{x+3} - 1$
- B. $f(x) = -\sqrt[3]{x-3} - 1$
- C. $f(x) = \sqrt[3]{x+3} - 1$
- D. $f(x) = \sqrt[3]{x+3} - 3$

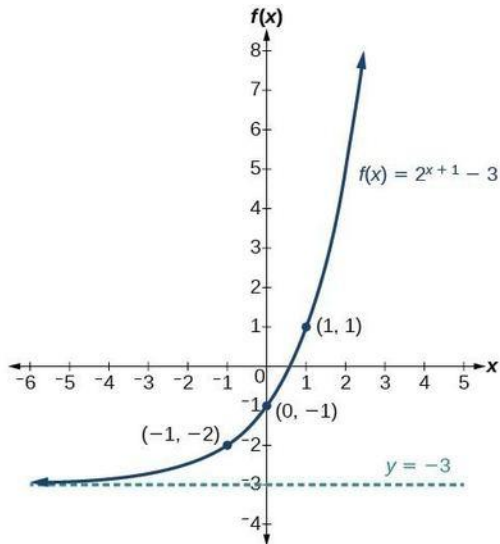
6. Which function matches the graph?



- A. $f(x) = \sqrt{x-1} + 3$
- B. $f(x) = -\sqrt{x-1} + 3$
- C. $f(x) = \sqrt{-x-1} + 3$
- D. $f(x) = -\sqrt{x-1} - 1$

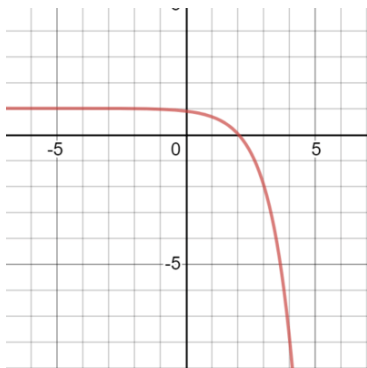
7. Use the graph of $y = 2^{x+1} - 3$.

What is the transformations from the parent function $y = 2^x$



- A. Translated right 1 and down 3
- B. Translated left 1 and down 3
- C. Translated up 1 and left 3
- D. Stretched by 2 and translated down 3

8. Use the graph of $y = -3^{x-2} + 1$. What is the transformations from the parent function $y = 2^x$.



- A. Translated left 2 and up 1
- B. Reflected about the x-axis, translated right 2 and up 1
- C. Reflected about the y-axis, translated left 2 and up 1
- D. Stretched by -3 and translated up 1

9. Write an absolute value function given the following transformations:

Vertical Stretch of 2

Horizontal shift left 1 unit

Vertical shift down 9 units

- A. $f(x) = |2x + 1| - 9$
- B. $f(x) = |2x - 1| - 9$
- C. $f(x) = 2|x + 1| - 9$
- D. $f(x) = 2|x - 1| - 9$

10. What are the transformations of $y = -2\sqrt{x-5}$ compared to the parent function?

- A. Shifted right 5, shifted down 2, and reflected over the x-axis
- B. Reflected over the x-axis, vertical stretch, and shifted right 5
- C. Reflected over the x-axis, vertical stretch, and shifted left 5
- D. Shifted right 5, shifted down 2

Week 2

Identifying Characteristics of Rational Functions Graphs

Remember:

Domain (x-values), Range (y-values), Asymptotes (horizontal and vertical lines that the graph does not reach), Intercepts (Point on the x/y axis where the graph intersects).

Vertical Asymptote: Vertical line that the graph approaches but doesn't intersect.

Where $b(x) = 0$ (Value that makes the denominator = 0)

Horizontal Asymptote: Horizontal line that the graph approaches. At most one horizontal asymptote.

Horizontal Asymptote: $y = 0$ Degree of $a(x) < \text{Degree of } b(x)$

No Horizontal Asymptote: Degree of $a(x) > \text{Degree of } b(x)$

Horizontal Asymptote: *Leading Coefficient of* $\frac{a(x)}{b(x)}$ Degree of $a(x) = \text{Degree of } b(x)$

Example: Identify the characteristics below for each graph.

1. $f(x) = \frac{-2x+1}{x+1}$

Domain: $(-\infty, -1)$ and $(-1, \infty)$

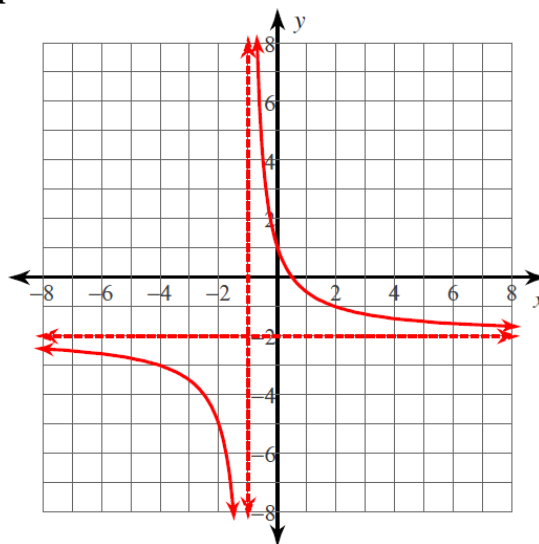
Range: $(-\infty, -2)$ and $(-2, \infty)$

Vertical Asymptote: $x = -1$

Horizontal Asymptote: $y = -2$

x-intercept: $(\frac{1}{2}, 0)$

y-intercept: $(0, 1)$



2. $f(x) = \frac{x+3}{x}$

Domain: $(-\infty, 0)$ and $(0, \infty)$,
All Reals $x \neq 0$

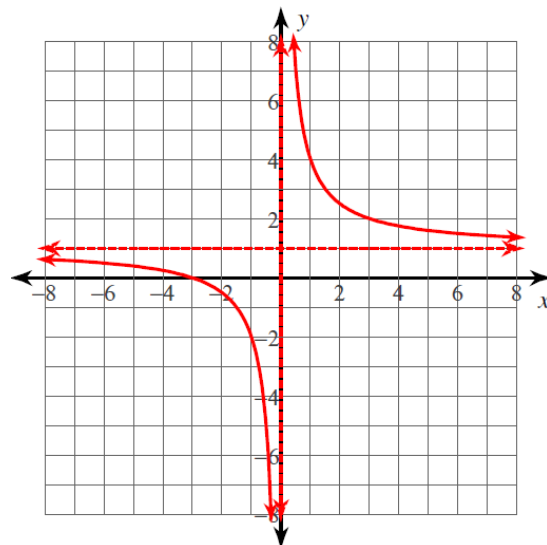
Range: $(-\infty, 1)$ and $(1, \infty)$,
All Reals $y \neq 1$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 1$

x-intercept: $(-3, 0)$

y-intercept: *none*



Your turn:

1. $f(x) = -\frac{4}{x^2-3x}$

Domain: _____

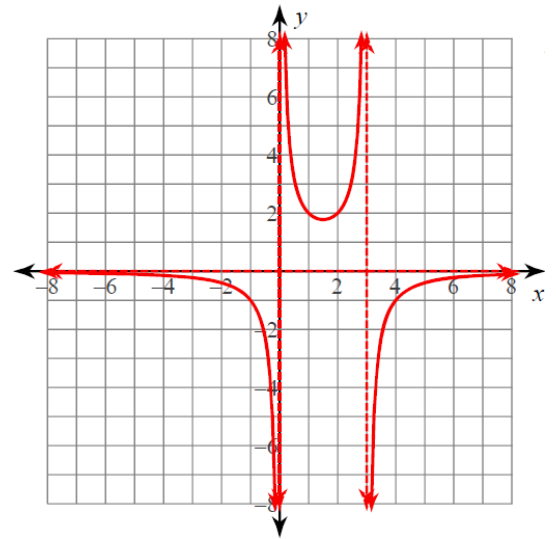
Range: _____

Vertical Asymptote: _____

Horizontal Asymptote: _____

x-intercept: _____

y-intercept: _____



2. $f(x) = \frac{x+2}{2x+6}$

Domain: _____

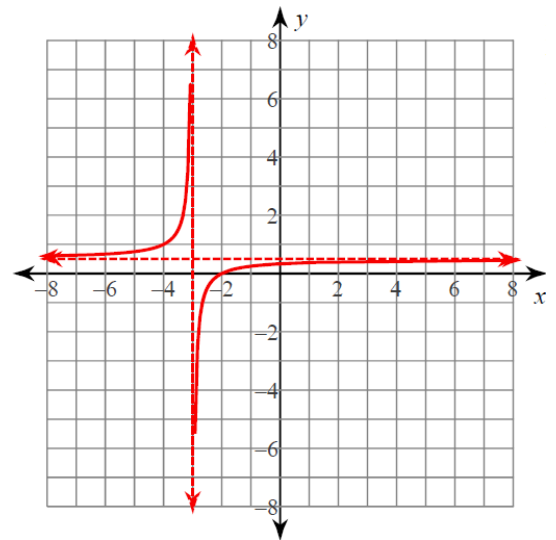
Range: _____

Vertical Asymptote: _____

Horizontal Asymptote: _____

x-intercept: _____

y-intercept: _____



3. $f(x) = \frac{3}{x-2}$

Domain: _____

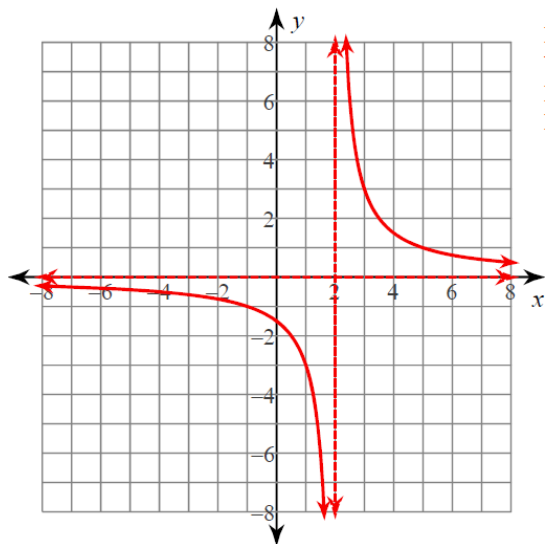
Range: _____

Vertical Asymptote: _____

Horizontal Asymptote: _____

x-intercept: _____

y-intercept: _____



TRANSFORMATIONS *Guide*

Translations	
$f(x+h)$	Shift left by h-units
$f(x-h)$	Shift right by h-units
$f(x)+k$	Shift up by k-units
$f(x)-k$	Shift down by k-units

Dilations		
$a \cdot f(x)$	when $ a > 1$	Vertical stretch by "a"
$a \cdot f(x)$	when $ a < 1$	Vertical shrink by "a"

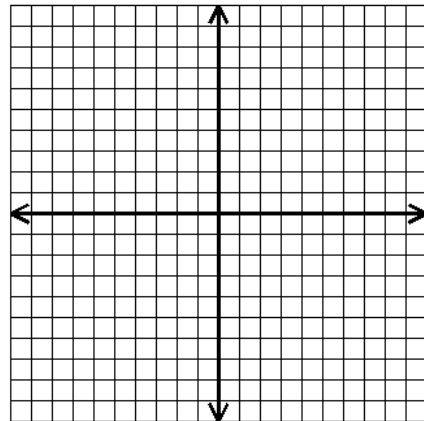
Reflections	
$-f(x)$	Reflection across the x-axis

1. $y = -|x+1| - 4$

Describe the transformations:

Domain: _____ Range: _____

Find where $-|x+1| - 4 \geq -6$ _____

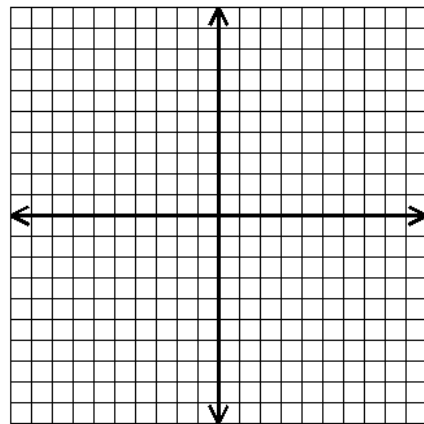


2. $y = 2(x+2)^3 - 1$

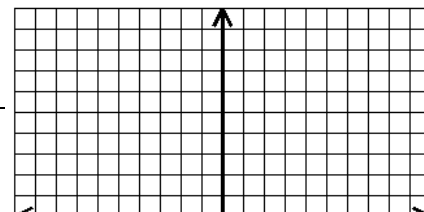
Describe the transformations:

Domain: _____ Range: _____

Find where $2(x+2)^3 - 1 < 1$ _____



3. $y = \frac{1}{x+3} + 1$



Describe the transformations:

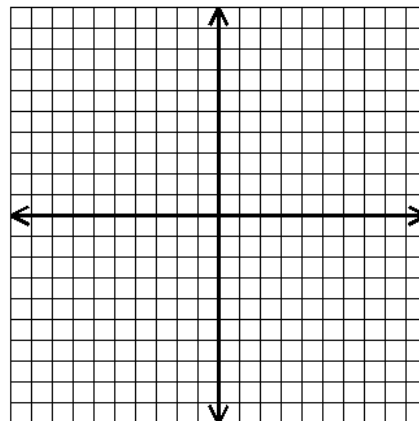
Domain: _____ Range: _____

4. $y = -\sqrt{2-x} - 2$

Describe the transformations:

Domain: _____ Range: _____

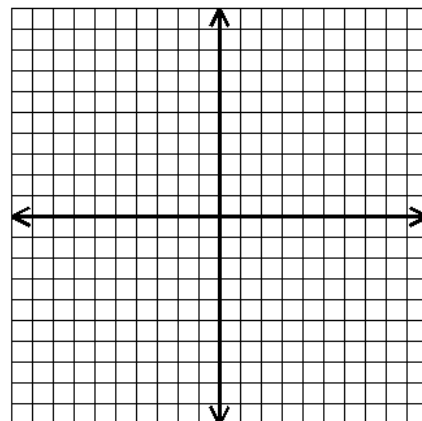
Find where $-\sqrt{2-x} - 2 \geq -4$ _____



5. $y = 3\sqrt{x-2} - 4$

Domain: _____ Range: _____

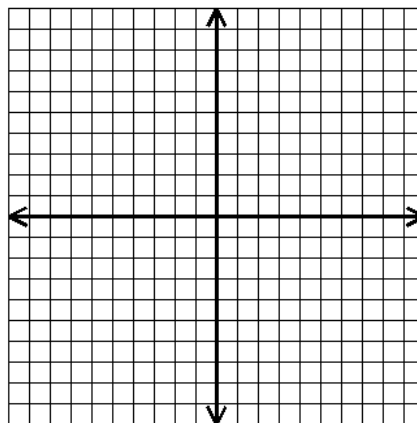
Find where $3\sqrt{x-2} - 4 > 2$ _____



6. $y = 2(x+1)^2 - 3$

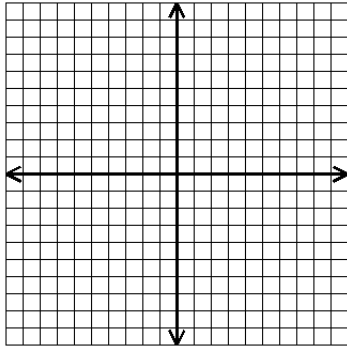
Describe the transformations:

Domain: _____ Range: _____

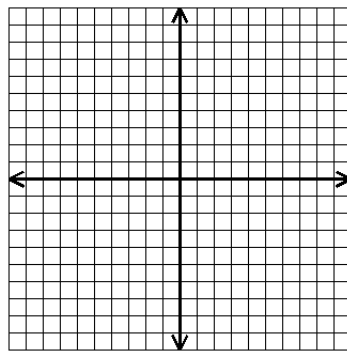


Graph each of the quadratic functions.

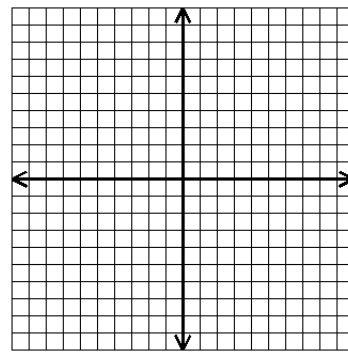
1. $y = -x^2 + 3$
 D: _____ R: _____



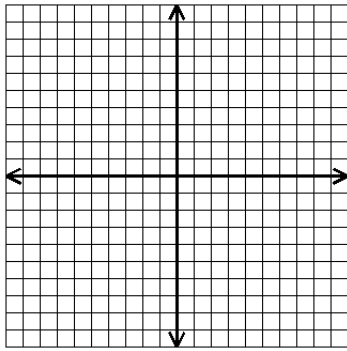
2. $y = (x - 1)^2 - 2$
 D: _____ R: _____



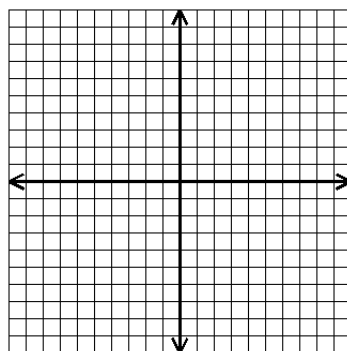
3. $y = (x + 2)^2 + 1$
 D: _____ R: _____



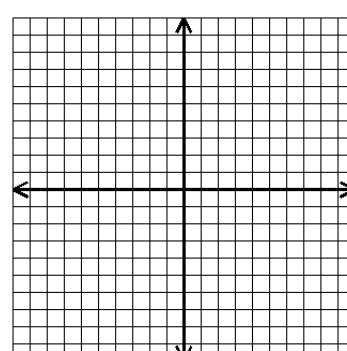
4. $y = 2(x - 2)^2$
 $y = -(x + 4)^2 + 1$
 D: _____ R: _____



6. $y = \frac{1}{2}x^2 + 3$
 D: _____ R: _____



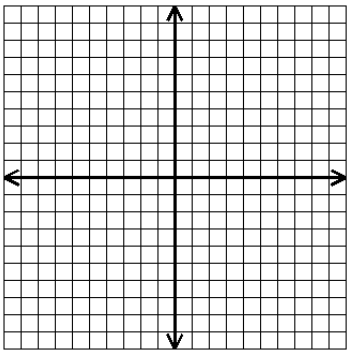
5. _____
 D: _____ R: _____



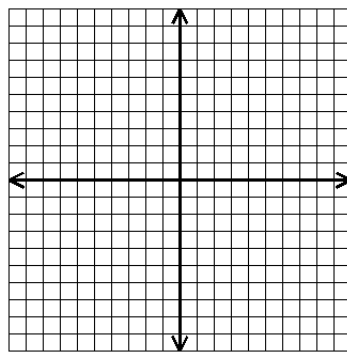
5.

7. $y = 2(x - 1)^2$

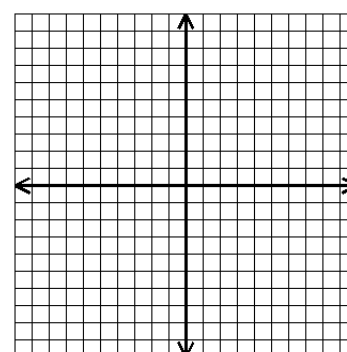
8. $y = (x + 3)^2 - 4$
 D: _____ R: _____



9. $y = \frac{1}{2}(x + 2)^2$
 D: _____ R: _____



_____ D: _____ R: _____



Write the equation for each of the following:

10. Start with the graph of $y = x^2$. Stretch it by a factor of 2 and shift it 2 units to the right. _

11. Start with the graph of $y = x^2$. Shrink it by a factor of $\frac{1}{2}$, then shift it 2 units to the right and 1 unit up.

12. Start with the graph of $y = x^2$. Shift it 5 units to the left. _

13. Start with the graph of $y = x^2$. Reflect it across the x-axis. Stretch it by a factor of 2. Shift right 3 units and up 1 unit.

Solving rational equations:

Remember that you need to have common denominators when adding and subtracting rational expressions.

1. Factor : Denominators
2. Find the LCD of all expressions on both sides
3. Multiply each term on both sides by the LCD, this will eliminate the fractions from the equation.
4. Solve the remaining equation left.
5. Check for extraneous roots (values that make the denominator undefined)

Example $\frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{x^2 - 13x + 42}$

$$9(x-6) - 7(x-7) = 13$$

$$2x = 18$$

LCD: $(x-7)(x-6)$ $\frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{(x-7)(x-6)}$

$$9x - 54 - 7x + 49 = 13$$

$$x = 9$$

$$2x - 5 = 13$$

What Law Firm Is Famous For Sneaky Legal Tricks?

Solve the equation, then find your answer. Write the letter of the answer in each box with the exercise number. If the answer has a ● shade in the box instead of writing a letter in it.

1 $\frac{x}{5} + \frac{x+4}{3} = 4$	2 $\frac{a}{7} - \frac{a-5}{2} = 3$	Answers S -4 O $\frac{7}{2}$ I $\frac{14}{3}$ ● $-\frac{10}{7}$ Y 16 H 5 W 8 R $-\frac{18}{5}$ A 3 H 7 T $\frac{10}{3}$ C $-\frac{7}{5}$ E $\frac{21}{2}$ H $\frac{8}{3}$ ● 12 D $\frac{42}{5}$
5 $\frac{3}{2y} + \frac{3}{5y} = \frac{7}{10}$	4 $\frac{2}{3c} = \frac{5+c}{4c} - \frac{5}{12}$	
5 $\frac{n}{3} - \frac{n-11}{6} = \frac{9}{2}$	6 $\frac{x-8}{5x} + \frac{3}{10} = \frac{x+2}{4x}$	
7 $\frac{5w}{2w+8} - \frac{1}{w+4} = 1$	8 $\frac{4}{5} + \frac{b}{b-2} = \frac{3b+8}{5b-10}$	
9 $\frac{2}{x+3} + \frac{7}{x-3} = \frac{2x+5}{x^2-9}$	10 $\frac{t+9}{t^2-4t} - \frac{3}{t} = \frac{1}{t-4}$	
11 $\frac{10k}{3k+15} = \frac{4}{k+5} + 2$	12 $\frac{x+3}{x} - \frac{x+1}{x+4} = \frac{5}{x}$	

6 11 12 11 5 9 2 10 11 11 7 10 3 1 9 3 8 6 9 10 4 12 11

Solving Radical Equations:

1. Isolate the radical in the equation (the term with the rational exponent)
2. Raise both sides of the equation to the reciprocal power

Ex. #1 $6 + \sqrt{3x + 1} = 11$

$$\sqrt{3x + 1} = 5$$

Square both sides

$$(\sqrt{3x + 1})^2 = 5^2$$

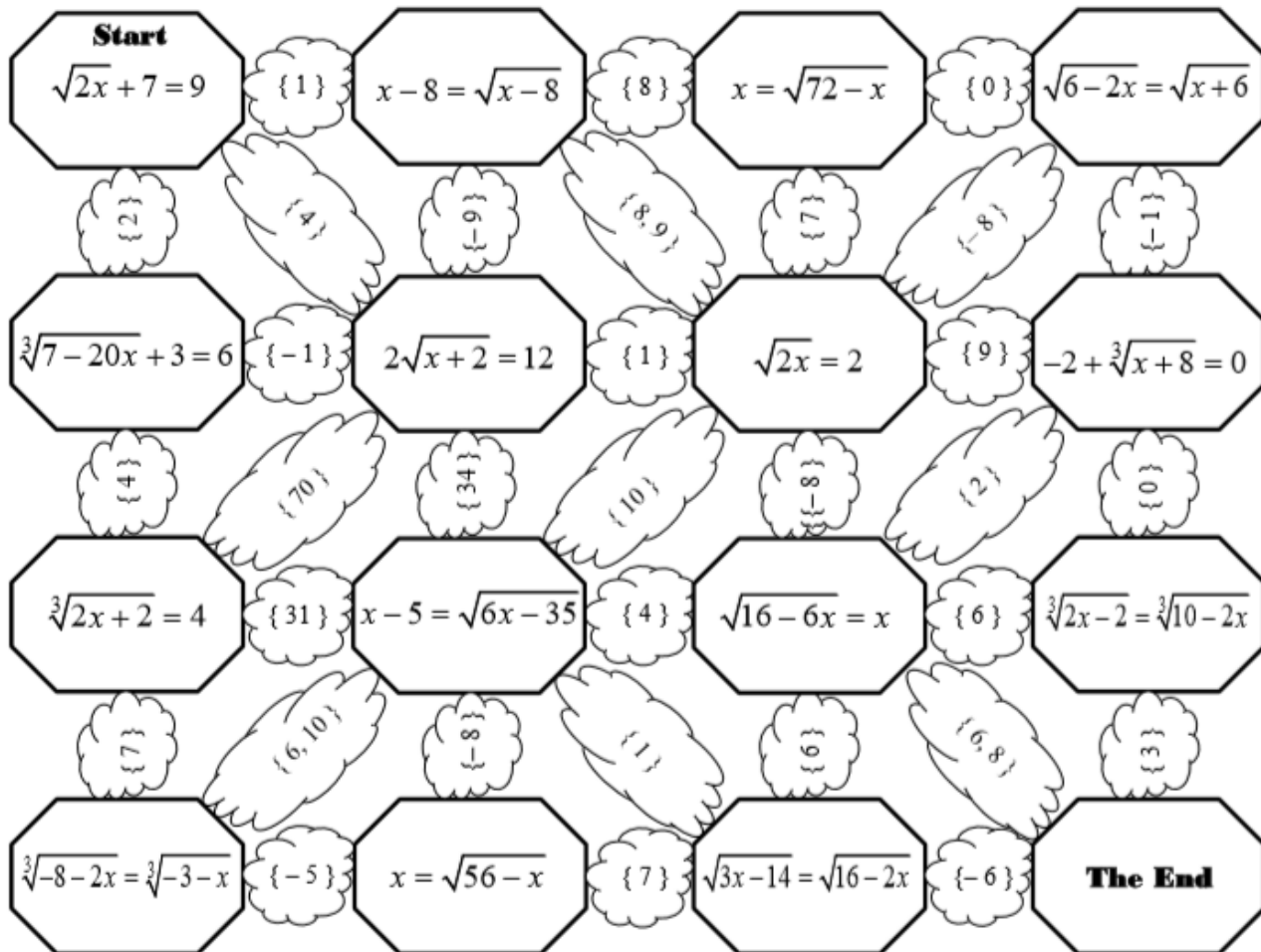
$$3x + 1 = 25$$

$$3x = 24$$

$$x = 8$$

Check your answer

Solving Radical Equations Maze



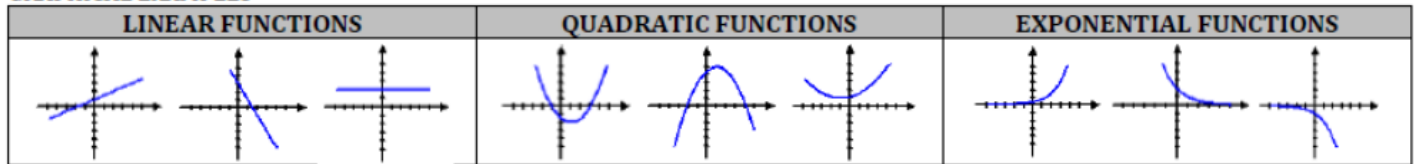
Week 3

- All.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.
- All.10 The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

Day 1: Curve of Best Fit

Here are 3 of the functions that we've covered and examples of what they look like:

GRAPHICAL EXAMPLES

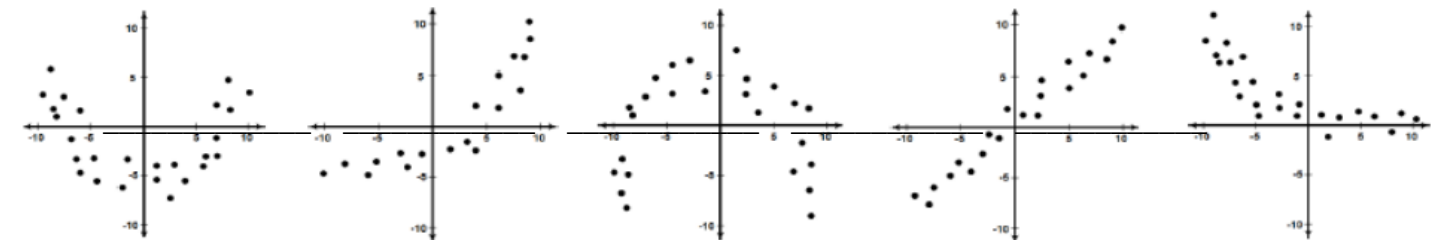


- Which function always has an asymptote? _____
- Which function will always have an absolute maximum or minimum value? _____
- Which function(s) can have end behavior that approaches a number? _____
- Circle one of the following for each: The given function will have at most _____ end behaviors that approach infinity or negative infinity.

○ Linear functions:	zero	one	two	no
○ Quadratic functions:	zero	one	two	no
○ Exponential functions:	zero	one	two	no

Try this one:

1. Graphically identify which type of function model might best represent each scatter plot.



Here are the parent functions of each:

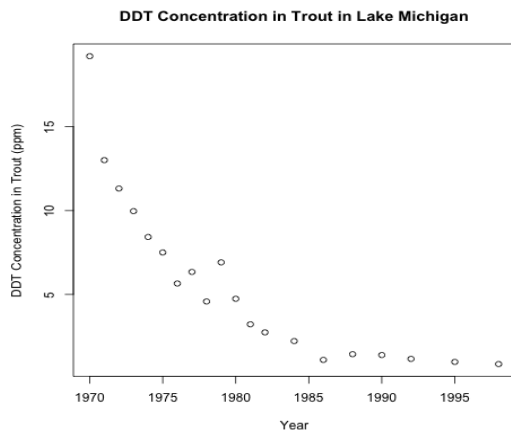
Linear	Quadratic	Exponential
$y = mx + b$	$y = ax^2 + bx + c$	$y = a^x$
Notice that for one unit increase in x , y increases/decreases by a constant amount.	Notice that as x increases, y increases/decreases at an increasing rate. This happens in both directions.	Notice that as x increases, y either increases at an increasing rate or it levels out to a number (look for an asymptote).

Determine whether the following table of values are Linear, Quadratic, Exponential or None of These.

1).	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>t</th><th>$P(t)$</th></tr> </thead> <tbody> <tr><td>-2</td><td>.111</td></tr> <tr><td>-1</td><td>.333</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>2</td><td>9</td></tr> <tr><td>3</td><td>27</td></tr> </tbody> </table>	t	$P(t)$	-2	.111	-1	.333	0	1	1	3	2	9	3	27	2).	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>w</th><th>$A(w)$</th></tr> </thead> <tbody> <tr><td>-2</td><td>8</td></tr> <tr><td>-1</td><td>2</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>3</td><td>18</td></tr> </tbody> </table>	w	$A(w)$	-2	8	-1	2	0	0	1	2	2	8	3	18	3).	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>x</th><th>$f(x)$</th></tr> </thead> <tbody> <tr><td>-2</td><td>4</td></tr> <tr><td>-1</td><td>2</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>.5</td></tr> <tr><td>2</td><td>.25</td></tr> <tr><td>3</td><td>.125</td></tr> </tbody> </table>	x	$f(x)$	-2	4	-1	2	0	1	1	.5	2	.25	3	.125	4).	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>t</th><th>$x(t)$</th></tr> </thead> <tbody> <tr><td>-2</td><td>4</td></tr> <tr><td>-1</td><td>16</td></tr> <tr><td>0</td><td>18</td></tr> <tr><td>1</td><td>17</td></tr> <tr><td>2</td><td>15</td></tr> <tr><td>3</td><td>19</td></tr> </tbody> </table>	t	$x(t)$	-2	4	-1	16	0	18	1	17	2	15	3	19	5).	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>x</th><th>$y(x)$</th></tr> </thead> <tbody> <tr><td>-2</td><td>-5</td></tr> <tr><td>-1</td><td>-3</td></tr> <tr><td>0</td><td>-1</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>3</td></tr> <tr><td>3</td><td>5</td></tr> </tbody> </table>	x	$y(x)$	-2	-5	-1	-3	0	-1	1	1	2	3	3	5
t	$P(t)$																																																																														
-2	.111																																																																														
-1	.333																																																																														
0	1																																																																														
1	3																																																																														
2	9																																																																														
3	27																																																																														
w	$A(w)$																																																																														
-2	8																																																																														
-1	2																																																																														
0	0																																																																														
1	2																																																																														
2	8																																																																														
3	18																																																																														
x	$f(x)$																																																																														
-2	4																																																																														
-1	2																																																																														
0	1																																																																														
1	.5																																																																														
2	.25																																																																														
3	.125																																																																														
t	$x(t)$																																																																														
-2	4																																																																														
-1	16																																																																														
0	18																																																																														
1	17																																																																														
2	15																																																																														
3	19																																																																														
x	$y(x)$																																																																														
-2	-5																																																																														
-1	-3																																																																														
0	-1																																																																														
1	1																																																																														
2	3																																																																														
3	5																																																																														
	_____		_____		_____		_____		_____																																																																						

Complete the following given the scatterplots shown:

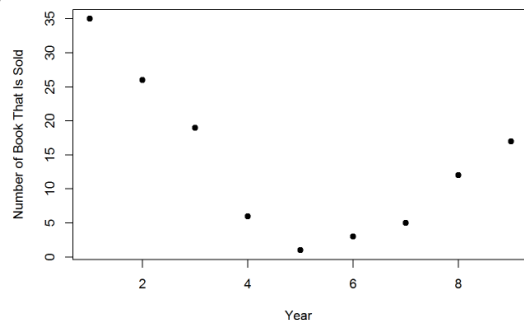
1). The following graph shows the concentration of DDT (a synthetic organic compound used in some pesticides) in parts per million was found in Trout in Lake Michigan over a series of years.



- 1). Based off of the graph, what type of regression equation should be used? (linear, quadratic, or exponential).

- 2). Based on this graph, approximately what would be the concentration (in ppm) in 1977? _____
- 3). Predict what year the concentration (in ppm) the concentration would be 2. _____

2). The number of books sold by a local bookstore based on the years since they opened in 2001 is shown below



- 1). Based off of the graph, what type of regression equation should be used? (linear, quadratic, or exponential).

- 2). Based on this graph, approximately How many books were sold in 2004? _____
- 3). Predict what year(s) the store sold 10,000 books.

Create a story that explains why the book stores sales acted like they did from 2001-2010.

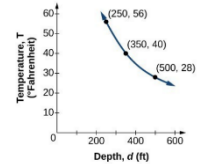
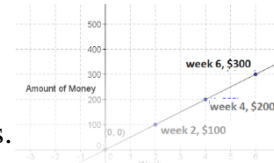
Day 2: Direct Variation

There are 3 different types of variation problems:

Direct Variation: $y = kx$ These are linear so as x increases y increases.

Inverse Variation: $y = \frac{k}{x}$ These are rational. As x increases y decreases (but not linearly)

Joint Variation: $y = kxz$ This one is just a “double direct”



Writing variation problems:

When writing variation (or proportionality) problems the variables can change. Therefore direct variation could be $A = kh$, inverse variation could be $I = \frac{k}{d}$, and joint variation could be $A = kPI$. However, the one variable that doesn't change is **k** ! This is called the constant of variance (or proportionality).

Direct Variation:

I. Writing the Model

Ex: H varies directly as M.

Step 1: Right when you see “**H varies**”... you automatically write “ **$H = k$** ”

Step 2: If it say “**directly as M**”... you put the “**M**” directly next to the **k** : “ **$H = kM$** ”

Ex: The amount of money that you make varies directly as the number of hours that you work.

Step 1: $A = k$

Step 2: $A = kh$

Try: At a constant speed, the total distance that you travel varies directly with the number of hours that are traveling. _____

II. Finding the Constant of Variance or Proportionality

Ex: Z varies directly with Y. If $Z = 6$ when $Y = 8$ find the constant of variance for this situation.

Step 1: Write the equation: $Z = kY$

Step 2: Plug in what is given: $6 = k(8)$

Step 3: Solve for k : $\frac{6}{8} = \frac{k(8)}{8}$ so $k = \frac{3}{4}$

Try: B varies directly with j. If $B = 100$ when $j = 5$ find the constant of variance for this situation.

Step 1: _____

Step 2: _____

Step 3: _____

Challenge: The distance (in miles) that your car can travel is directly proportional to the gallons of gas that you put into your car. If your car traveled 308 miles and you put 14 gallons of gas in it, what is the constant of proportionality? Can you interpret what k means in this situation?

III. Finding the Equation and Determining Other Values.

Ex: If y is directly proportional to x and $y = 40$ when $x = 2$ find x when $y = 100$.

Step 1: Write the equation: $y = kx$

Step 2: Find k : $40 = k(2)$ so $k = 20$

Step 3: Write the equation with k : $y = 20x$

Step 4: Answer the question: $100 = 20x$ so $x = 5$

Try: If m varies directly as t and $m = 15$ when $t = 3$ find m when $t = 12$,

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Day 2 Assignment:

1) If z is directly proportional to r and $z = 5$ when $r = 10$, find r when $z = 7$.

2) If y varies directly as t and $y = 5$ when $t = 2$, find y when $t = 10$.

3) When a person swims underwater the pressure in his/her ears varies directly with the depth at which he/she is swimming. At 10ft the pressure is about 4.4 pounds per square inch (psi).

A). Find the equation that represents the situation.

B). Find the pressure if the depth is 100ft.

C). It is unsafe for amateur divers to swim where the pressure is more than 65 psi. How deep can they swim safely?

4) The force (F) needed to stretch a spring by a distance x is given by the equation $F=k \cdot x$, where k is the spring constant (measured in Newtons per centimeter, N/cm). If a 12-Newton force stretches a certain spring by 10 cm, calculate:

a. The spring constant, k

b. The force needed to stretch the spring by 7 cm

c. The distance the spring would stretch with a 23-Newton force

Day 3: Inverse and Joint Variation (GOOD NEW!!!! Inverse and Joint Variation work EXACTLY like direct except the initial equation is different).

Joint Variation: (double direct)

Ex: If y varies jointly as x and z and $y = 12$ when $x = 3$ and $z = 2$, write an equation that represents this situation.

Step 1: REMEMBER when it says “ y varies” you write $y = k$

Step 2: it says “jointly as x and z ” (and since jointly is a double direct), the x and z go directly next to the k . So it's: $y = kxz$

Ex: I varies jointly as P and T . If $I = 1200$ when $P = 5000$ and $T = 3$, find I when $P = 7500$ and $T = 4$.

Step 1: $I = kPT$

Step 2: $1200 = k(5000)(3)$ so: $1200 = k(15000)$ and $k = 0.8$

Step 3: $I = 0.8PT$

Step 4: $I = 0.8(7500)(4) = 24000$

Try These:

- 1) The cost c of materials for a deck varies jointly with the width w and the length l . If $c = \$470.40$ when $w = 12$ and $l = 16$, find the cost when $w = 10$ and $l = 25$.
- 2) The value of real estate V varies jointly with the neighborhood index N and the square footage of the house S . If $V = \$376,320$ when $N = 96$ and $S = 1600$, find the value of a property with $N = 83$ and $S = 2150$.
- 3) The number of gallons g in a circular swimming pool varies jointly with the square of the radius r^2 and the depth d . If $g = 754$ when $r = 4$ and $d = 2$, find the number of gallons in the pool when $r = 3$ and $d = 1.5$. (note: in step 1 you need to use r^2)
- 4) The simple interest “ I ” earned by an investment varies jointly as the rate “ r ” and the amount of time “ t ”. In 5 years, an investment earns \$1540 at a rate of 5%. How much will the investment earn in 10 years at 4%.

Inverse Variation: (Instead of going directly next to the k , it goes underneath of it)

Ex: y varies inversely as x and $y = 5$ when $x = 10$, write an equation that represents this situation.

Step 1: REMEMBER when it says “ y varies” you write $y = k$

Step 2: it says “inversely as x ” so you write the x underneath of the k . So it's $y = \frac{k}{x}$

Ex: The time it takes to travel a fixed distance varies inversely with the speed traveled. If it takes Pam 40 minutes to bike to the secret fishing spot at 9 miles per hour, what is the equation that represents this situation? How long will it take if she rides 12 miles per hour?

(Keep in mind that this problem works with miles per hour so “40 minutes” is actually $\frac{2}{3}$ hours)

Step 1: $t = \frac{k}{s}$

Step 2: $\frac{2}{3} = \frac{k}{9}$ so $18 = 3k$ and $k = 6$

Step 3: $t = \frac{6}{s}$

Step 4: $t = \frac{6}{12} = \frac{1}{2}$ hours

Try these:

- 1) Illuminance is the intensity of light that the eye perceives. As you move closer to or farther from a light source, the illuminance varies inversely as the square of the distance “ d ” from the source. At 105ft from a baseball pitcher, a sports photographer’s light meter registers an illuminance of 30 foot candles. Determine the meters reading at a distance of 50ft from the pitcher.
- 2) The time to complete a project varies inversely with the number of employees. If 3 people can complete the project in 7 days, how long will it take 5 people?
- 3) The time needed to travel a certain distance varies inversely with the rate of speed. If it takes 8 hours to travel a certain distance at 36 miles per hour, how long will it take to travel the same distance at 60 miles per hour?
- 4) The number of revolutions made by a tire traveling over a fixed distance varies inversely to the radius of the tire. A 12-inch radius tire makes 100 revolutions to travel a certain distance. How many revolutions would a 16-inch radius tire require to travel the same distance?

Day 4: Combining Variation Types

Sometimes variables vary in a combination of different ways with other variables. Here are a few examples:

- z varies inversely with x but directly with y : $z = \frac{ky}{x}$
- y is directly proportional to the square root of m : $y = k\sqrt{m}$
- x varies jointly with r and s but inversely with the square of t : $x = \frac{krs}{t^2}$

Let's take today to make sure that days 1-3 are completed and then try the following:

- 1) To build a sound wall along the highway, the amount of time t needed varies **directly** with the number of cement blocks c needed and **inversely** with the number of workers w . A sound wall made of 2400 blocks, using six workers takes 18 hours to complete. How long would it take to build a wall of 4500 blocks with 10 workers? HINT: $t = \frac{kc}{w}$
- 2) The time needed to paint a fence varies directly with the length of the fence and indirectly with the number of painters. If it takes five hours to paint 200 feet of fence with three painters, how long will it take five painters to paint 500 feet of fence?
- 3) The time to prepare a field for planting is inversely proportional to number of people who are working. A large field can be prepared by five workers in 24 days. In order to finish the field sooner, the farmer plans to hire additional workers. How many workers are needed to finish the field in 15 days?
- 4) An egg is dropped from the roof of a building. The distance it falls varies directly with the square of the time it falls. If it takes $\frac{1}{2}$ second for the egg to fall eight feet, how long will it take the egg to fall 200 feet?

- 5) The number of hours needed to assemble computers varies directly as the number of computers and inversely as the number of workers. If 4 workers can assemble 12 computers in 9 hours, how many workers are needed to assemble 48 computers in 8 hours?
- 6) The weight of a person varies inversely as the square of the distance from the center of the earth. If the radius of the earth is 4000 miles, how much would a 180 pound person weigh, 2000 miles above the surface of the earth?
- 7) The strength of a rectangular beam varies jointly as its width and the square of its depth. If the strength of a beam three inches wide by 10 inches deep is 1200 pounds per square inch, what is the strength of a beam four inches wide and six inches deep?

Day 5: Regression and Variation Problems on the SOL

1. The amount of work (W) done when lifting an object varies jointly with the mass of the object (m) and the distance the object is lifted (D). Which equation models this relationship?

A. $W = \frac{k}{MD}$

B. $W = \frac{kM}{D}$

C. $W = kMD$

D. $W = \frac{kD}{M}$

2. **Might need a calculator but try to narrow down the answers!**

Madison deposited \$1,000 into a savings account that compounds interest yearly. After her initial deposit, Madison did not withdraw or deposit any money from this account. The table below shows the amount in her savings account over a period of years.

Amount in Savings Account

Number of Years After the Deposit	Amount in Savings
2	\$1,123.60
4	\$1,262.48
6	\$1,418.52
8	\$1,593.85
10	\$1,790.85

Using the exponential curve of best fit, which is closest to the expected amount in the savings account 30 years after the time Madison deposited the initial \$1,000?

A. \$2,854

B. \$3,291

C. \$5,743

D. \$16,854

3. If y varies inversely as the square root of x , what is the constant of proportionality if $y = 16$ when $x = 4$?

A. 4

B. 8

C. 32

D. 64

4. The volume of a cone (V) varies jointly with its heights (h) and the square of its radius (r). If k is the constant of proportionality, which of the following equations represents the correct relationship between volume, radius, and height?

A. $V = k(rh)^2$

B. $V = \frac{kr^2}{h}$

C. $V = \frac{k}{r^2h}$

D. $V = kr^2h$

5. **(Remember that an exponential has the form $y = ab^x$ where a is the initial value and b is the rate or r from a geometric sequence)**

Jessica paid \$23,000 for her car and kept a record of its value.

Number of Years (x)	Value (in dollars) (y)
0	23,000
1	20,000
2	16,000
3	14,000
4	12,000
5	10,000

Assuming the relationship is exponential, which equation best models the curve of best fit for the data?

- A. $y = 21,000(1.20)^x$ B. $y = 22,300(2.60)^x$ C. $y = 23,100(0.85)^x$ D. $y = 23,500(0.70)^x$

6. **Again, you could use a calculator. However, notice that it's decay. It has to decrease less and less each time. From 3 to 4 it decreases by 8 so from 4 to 5 it has to decrease by less than that. See which answers are possible.**

A scientist obtained a sample that contained 80 grams of radioactive Barium-122 that decays exponentially over time. The amount of Barium-122 that remained in the sample at observed times is shown in the table.

Radioactive Decay of Barium-122

Time (minutes)	Mass of Remaining Barium-122 (grams)
0	80.0
1	56.6
2	40.0
3	28.3
4	20.0

If the radioactive decay continue at the same rate, which is closest to the amount of the sample of Barium-122 remaining at 5 minutes?

- A. 8.3 grams B. 10.0 grams C. 11.7 grams D. 14.1 grams

7. The time it takes to travel a given distance varies inversely as the average rate of travel. Averaging 42 miles per hour, it takes Andre 5 hours to drive to Donnegal. If it took him 4 hours and 20 minutes to reach Donnegal on his last trip, what was his average rate of travel?

- F. 49.4 mi/hr G. 46.7 mi/hr H. 36.4 mi/hr J. 48.5 mi/hr

8. A farmer pumps water from an irrigation well to water his field. The time it takes to water the field varies inversely with the rate at which the pump operates. It takes 20 hours to water the field when the pumping rate is 600 gallons per minute. If the adjusts the pump so that it pumps at a rate of 400 gallons per minute, how long will it take to water the field?

- F. 40 hours G. 30 hours H. 12.5 hours J. 15 hours

Answers: 1) C, 2) C, 3) C, 4) D, 5) F, 6) D, 7) J, 8) G